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$$\therefore \frac{2}{u}\frac{\partial u}{\partial x} = -\frac{1}{s-x} + \frac{1}{x+y-s}. \quad \therefore \frac{\partial u}{\partial x} = \frac{u}{2}\,\frac{2s-2x-y}{(s-x)(x+y-s)}\ldots(2),$$

$$\text{and } \frac{\partial u}{\partial y} = \frac{u}{2}\,\frac{2s-2y-x}{(s-y)(x+y-s)}\ldots(3).$$

Equating (2) and (3) to 0, we have $2s-2x-y=0$, $2s-2y-x=0$, whence $x=\frac{2}{3}s$, $y=\frac{2}{3}s$, $z=2s-x-y=\frac{2}{3}s$.

Hence the triangle is equilateral; and there is evidently a maximum.

309. Proposed by S. G. BARTON, Ph. D., Clarkson School of Technology.

In practical problems involving maxima and minima, it is really the greatest and least values of the function which are desired. Show why we can assume that the maximum is the greatest value and the minimum the least value under the conditions.

No solution of this problem has been received.

310. Proposed by C. N. SCHMALL, New York City.

Evaluate $\int_0^\pi \frac{dx}{1-2a\cos x+a^2}$. Edwards' *Integral Calculus for Beginners*, page 131, ex. 9, (iii). The answer given is $\frac{\pi}{1-a^2}$. Is this a complete answer to the question?

I. Solution by H. PRIME, Boston, Mass.

$$\int_0^\pi \frac{dx}{1+a^2-2a\cos x} = \left[\frac{2}{\pm(1-a^2)}\tan^{-1}\left(\pm\frac{1+a}{1-a}\tan\frac{x}{2}\right)\right]_0^\pi$$

$$= \frac{(2n+1)\pi}{\pm(1-a^2)} - \frac{2n\pi}{\pm(1-a^2)} = \frac{\pi}{\pm(1-a^2)}.$$

It should be noted that if $a=\pm1$, the function $1/(1+a^2-2a\cos x)$ becomes infinite for $x=0$ or $x=\pi$. Hence, in this case, the integral has no meaning for the required limits, as they do not exist. The result also shows this for $\pi\div(1-a^2)=\infty$, when $a=\pm1$.

Also solved by J. Scheffer, Francis Rust, A. M. Harding, and V. M. Spunar.

311. Proposed by WILMER THOMPSON, Senior, Drury College.

Solve the differential equation,

$$\left(\frac{dy}{dx}\right)^3 + x^3 = ax\left(\frac{dy}{dx}\right).$$

[From Forsythe's *Differential Equations*, p. 47.]

I. Solution by **W. W. BEMAN**, Professor of Mathematics, University of Michigan, and **A. H. HOLMES**, Brunswick, Maine.

$p^3+x^3=axp$. Putting $p=xu$, $x=\dfrac{au}{1+u^3}$, $p=\dfrac{au^2}{1+u^3}$.

Further, $p=\dfrac{dy}{du}\cdot\dfrac{du}{dx}=\dfrac{(1+u^3)^2}{a\,(1-2u^3)}\cdot\dfrac{dy}{du}$.

$\therefore\ \dfrac{dy}{du}=a^2\dfrac{(1-2u^3)u^2}{(1+u^3)^3}$. Integrating, $y+c=\dfrac{a^2}{6}\cdot\dfrac{1+4u^3}{(1+u^3)^2}$.

Eliminating u from this equation and $x=\dfrac{au}{1+u^3}$,

$$216(y+c)^3-36a^2(y+c)^2-108ax^3(y+c)+16a^3x^3+27x^6=0.$$

II. Solution by **M. E. GRABER, A. M.**, Heidelberg University, Tiffin, Ohio.

Expressing the above equation in the form, $p^3-axp+x^3=0$, and solving for p by Cardan's Method, we obtain

$$p=\left(-\frac{x^2}{2}+\sqrt{+\frac{x^6}{4}-\frac{a^3\,x^3}{27}}\right)^{1/3}+\left(-\frac{x^3}{2}-\sqrt{+\frac{x^6}{4}-\frac{a^3\,x^3}{27}}\right)^{1/3}\ldots(1).$$

Assuming $-\dfrac{a^3\,x^3}{27}=-\dfrac{x^6}{4}\sin^2\theta\ldots(2)$, (1) becomes

$$p=(-x)\left[(\cos\tfrac{1}{2}\theta)^{2/3}+(\sin\theta)^{2/3}\right]\ldots(3),\ \text{and}\ x=\frac{4^{1/3}a}{3(\sin\theta)^{2/3}}\ldots(4).$$

From (4), $dx=-\dfrac{2.4^{1/3}a}{9}\,\dfrac{\cos\theta}{(\sin\theta)^{\frac{5}{3}}}d\theta$. By substituting in (1),

$$p=\frac{2.16^{2/3}\,a^2}{27}\left[\frac{(\cos\frac{1}{2}\theta)^{2/3}}{(2\sin\frac{1}{2}\theta\,\cos\frac{1}{2}\theta)^{2/3}}+\frac{(\sin\frac{1}{2}\theta)^{2/3}}{(2\sin\frac{1}{2}\theta\,\cos\frac{1}{2}\theta)^{2/3}}\right]\frac{(2\cos^2\frac{1}{2}\theta-1)}{(2\sin\frac{1}{2}\theta\,\cos\frac{1}{2}\theta)^{\frac{5}{3}}}$$

$$dy=\frac{2.16^{1/3}.a^2}{27.2^{\frac{7}{3}}}\left[\frac{2(\cos\frac{1}{2}\theta)^{1/3}}{(\sin\frac{1}{2}\theta)^{\frac{7}{3}}}-\frac{1}{(\sin\frac{1}{2}\theta)^{\frac{7}{3}}\,(\cos\frac{1}{2}\theta)^{\frac{5}{3}}}+\frac{2}{(\sin\frac{1}{2}\theta)^{\frac{5}{3}}\,(\cos\frac{1}{2}\theta)^{1/3}}\right.$$

$$\left.-\frac{1}{(\sin\frac{1}{2}\theta)^{\frac{5}{3}}\,(\cos\frac{1}{2}\theta)^{\frac{7}{3}}}\right]d\theta.$$

Placing $\tan\theta=t$, $d\theta=\frac{dt}{1+t^2}$, $\sin\theta=\frac{t}{(1+t^2)^{1/2}}$, and $\cos\theta=\frac{1}{(1+t^2)^{1/2}}$.

Integrating, we obtain,

$$y+c=-\tfrac{3}{4}\left[(\tan\tfrac{1}{2}\theta)^{-\frac{4}{3}}+(\tan\tfrac{1}{2}\theta)^{\frac{4}{3}}\right]\frac{2.16^{1/3}a^2}{27.2^{\frac{7}{3}}}=-\frac{a^2}{36}\left[\left(\tan\tfrac{1}{2}\theta\right)^{-\frac{4}{3}}+\left(\tan\tfrac{1}{2}\theta\right)^{\frac{4}{3}}\right]$$

from which, by substitution, the solution may be completed.

III. Solution by WILLIAM HOOVER, Athens, Ohio.

Let $dy/dx=p$, as usual; then $p^3-axp+x^3=0$.

The condition for equal values of p from this cubic is

$$\frac{p^6}{4}=\frac{a^3p^3}{27}, \text{ or } p=\frac{dy}{dx}=\frac{a}{3}\sqrt[3]{4}.$$

Integrating, $\frac{3y}{a\sqrt[3]{4}}=x+c$, or, $27y^3=4a^3(x+c)^3$.

Also solved by S. G. Barton.

MECHANICS.

255. Proposed by the late G. B. M. ZERR, Ph. D.

Assuming the resilience of volume of mercury to be constant at all depths and to be 54. 20×10^{10} in *C. G. S.* units and that a mile = 160933 centimeters. Find the depth of an ocean of mercury at a point where its density is double the surface density, 13.596.

Solution by B. F. FINKEL, Ph. D., Drury College.

Let x_0 be the length of a column of mercury of one square centimeter cross-section and uncompressed, such that its weight is sufficient to compress a cubic centimeter to half its volume, which is equivalent to doubling its density.

By Hooke's law, $p=E\frac{\Delta v}{v}=\frac{1}{2}E$, where E is the coefficient of elasticity. But $p=13.596gx_0$.

$\therefore x_0=\frac{E}{27.192g}$. Let Δx_0=the amount of compression of the column by virtue of its own weight, and let x=the length of any portion of the column measured from the surface downward, and let dx be an element of length of this column. Then the strain, s, in this element is